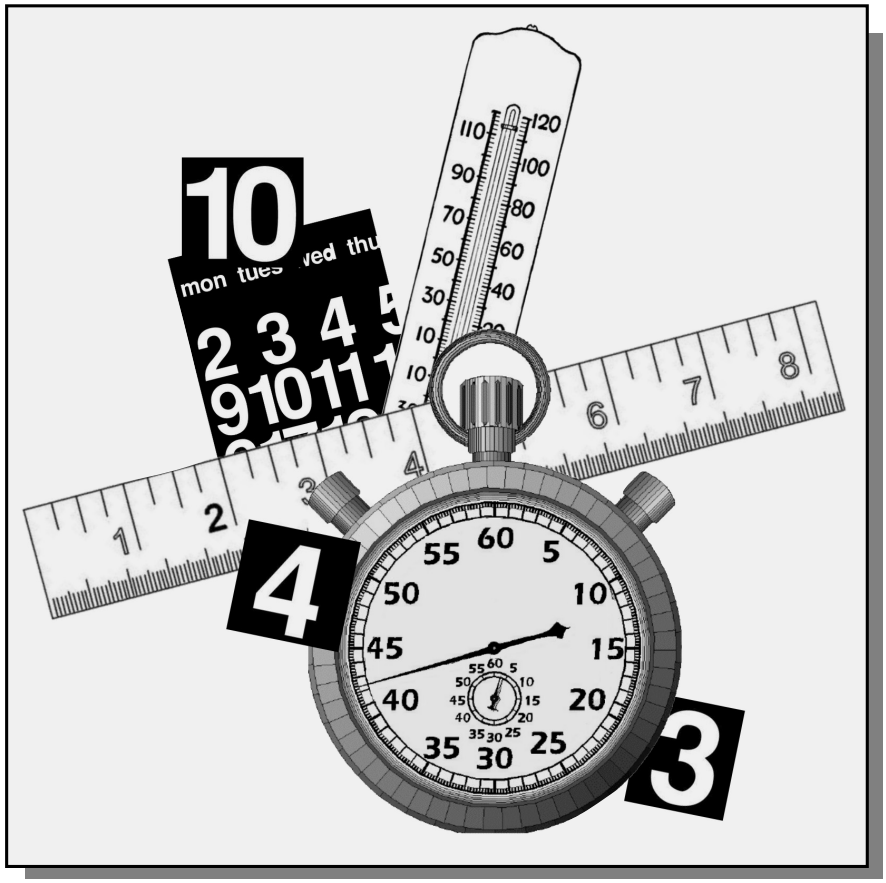
Intermediate Math Skills



Chapter 5

IMS: 76033 Revision: 030208

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

PROFESSIONAL CREDITS

Intermediate Math Skills

Chapter 5

Publisher and Managing Editor
Ronald E. Johnson, Ph.D.

Writers
Kermit Northcutt
David Browder

Technical Editor
David Browder

Content Editor / QA
Nancy Johnson

Production Manager
H. Michael Young

Page Composition
Debbie L. Dial
Kathy L. Dial
Pamela K. Hurst
H. Michael Young

Graphic Artists
Kathy L. Dial
Pamela K. Hurst
Kristin Odom
Lydia Seals
Don Ward
Dan White
H. Michael Young

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

CHAPTER 5
TABLE OF CONTENTS

PAGE

SECTION 1

1. RELATIONS 1-3
Range and Domain, Graphing on Cartesian Coordinate Planes

2. FUNCTIONS 5-8
Vertical Line Test, Cartesian Coordinate System

3. MORE FUNCTIONS 9-11
Value and Number of a Term, The Nth Term

4. GRAPHING FUNCTIONS 13-15
Cartesian Coordinate Planes . . . . . Ordered Pairs, Vertical Line Test

5. OTHER RELATIONS 17-18
Equations with Ordered Triplets

SECTION 2

6. POLYNOMIALS 19-20
Monomials, Binomials, Trinomials

7. BINOMIALS 21-23
Adding by Grouping Like Terms

8. ADD BI- and TRI-NOMIALS 25-27
Combining Like Terms, Adding Coefficients

PAGE

9. ADD TRINOMIALS 29-30
Combining Like Terms, Adding Coefficients

10. DISTRIBUTIVE PROPERTY . . .31-32
Alternative to PEMDAS for Simplifying Mathematical Expressions

SECTION 3

11. DISTRIBUTE FRACTIONS 33-34
Simplifying Fractions Containing Algebraic Expressions

12. SUBTRACT BINOMIALS 35-37
Using Commutative and Distributive Properties

13. SUBTRACT TRINOMIALS 39-40
Using Commutative and Distributive Properties

14. SUBTRACT MORE TRINOMIALS 41-43
Using Commutative and Distributive Properties

15. ADD & SUBTRACT POLYS . . .45-47
Using Commutative and Distributive Properties

MATHEMATICS CHARTS 49-51



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

"The study of Mathematics cannot be replaced by any other activity that will train and develop man's purely logical faculties to the same level of rationality."

—C. O. Oakley



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

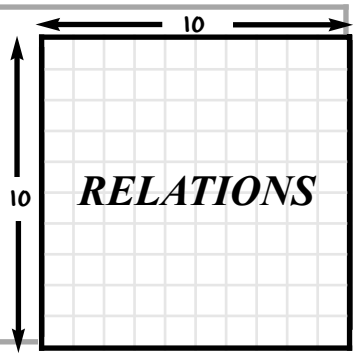
Intermediate Math Skills



Lesson 1

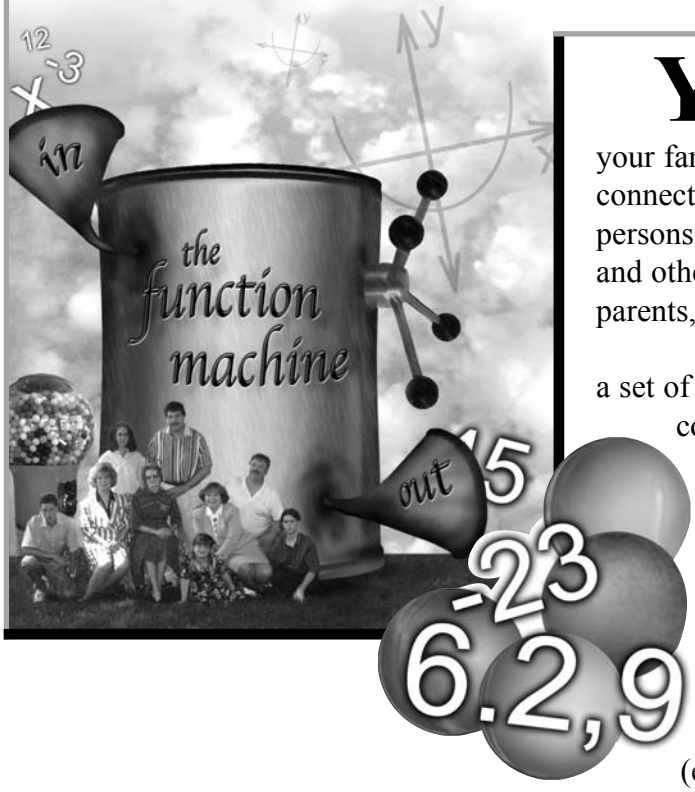
Graphs

Chapter 5



Section 1

Lessons 1-5



You probably consider a *relation* to be someone who is ***kin*** to you or part of your family. A relation means that there is a connection of some kind between two or more persons. In your family, the connection is that you and other members of your family have the same parents, grandparents or great grandparents.

In mathematics, a relation is defined as a set of ordered pairs (x, y). Usually, there is some connection between the two members of an ordered pair. This connection is often determined by an equation. In the equation, $y = x + 3$, y is considered a dependent variable and x is the independent variable. The x is considered independent because you can allow x to be any number that you choose. However, when you follow the rule (equation) the value of y depends on the value of x that you choose.

The set of values for the dependent variable (y) is called the ***range*** of the relation. The set of values for the independent variable (x) is known as the ***domain*** of the relation.

Follow along in the examples as Professor K shows you how to determine the *domain* and *range* of the *relations* shown.

VOCABULARY

Kin: a family member
Range: the set of all the possible values of the y-axis or dependent variable
Domain: the set of all possible values of the x-axis or independent variable
Ascending: increasing or going up

Example Set 1

Determine the domain and range of the set of ordered pairs.

{(1, 2), (3, 4), (5, 3), (2, 6), (0, 10)}

Step 1 values of x: 0, 1, 2, 3, 5
values of y: 2, 3, 4, 6, 10

Professor K lists the values of x and y in ***ascending*** order.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

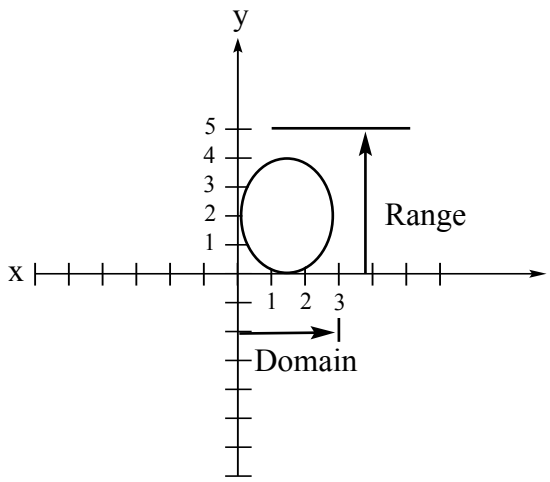
Step 2 Domain, D:{0, 1, 2, 3, 5}
Range, R: {2, 3, 4, 6, 10}

Professor K places the values of x in curly bracket set symbols and names this set the *Domain, D*. Professor K places the values of y in curly bracket set symbols and names this set the *Range, R*.

Example Set 2

Determine the domain and range of the relation graphed on the Cartesian coordinate plane below.





Step 1 minimum value of x: 0
maximum value of x: 3
$0 \leq x \leq 3$
Domain: $\{x \mid 0 \leq x \leq 3\}$

In Step 1, Professor K determines where the graph begins (zero) and ends (3) along the x-axis. Professor K realizes that the lowest value of x is zero, therefore, zero is the minimum value of x. Professor K also sees that 3 is the largest value of x. Therefore, 3 is the maximum value of x. Professor K sets up an inequality statement to define the values of x ($0 \leq x \leq 3$). Professor K states the Domain as $\{x \mid 0 \leq x \leq 3\}$.

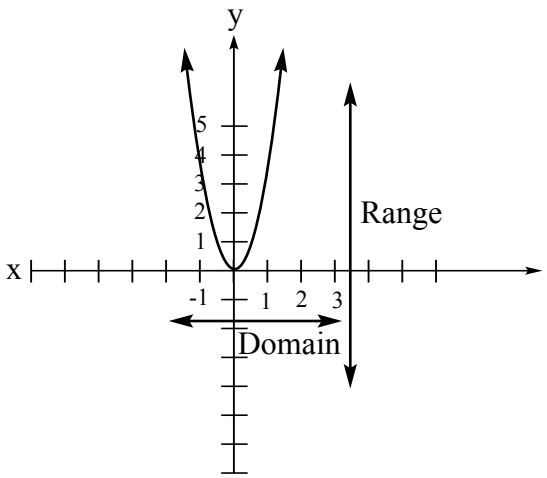
Step 2 minimum value of y: 0
maximum value of y: 4
$0 \leq y \leq 4$
Range: $\{y \mid 0 \leq y \leq 4\}$

In Step 2, Professor K determines where the graph begins (zero) and ends (4) along the y-axis. Professor K realizes that the lowest value of y is zero, therefore zero is the minimum value of y. Professor K also sees that 4 is the largest value of y. Therefore, 4 is the maximum value of y. Professor K sets up an inequality statement to define the values of y ($0 \leq y \leq 4$). Professor K states the Range as $\{y \mid 0 \leq y \leq 4\}$.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Example Set 3

Determine the domain and range of the relation graphed on the Cartesian coordinate plane below.



Step 1

minimum value of x: $-\infty$
maximum value of x: ∞
Domain: $\{x \mid -\infty < x < \infty\}$
$-\infty < x < \infty$

In Step 1, Professor K determines the graph has no beginning along the x-axis. Therefore, negative infinity is the lowest value of x and also the minimum value of x. Professor K also sees that there is no largest value of x. Therefore, positive infinity (∞) is the maximum value of x. Professor K sets up an inequality statement to define the values of x. Professor K states the Domain of the relation as $\{x \mid -\infty < x < \infty\}$.

Step 2

minimum value of y: 0
maximum value of y: ∞
$0 \leq y < \infty$
Range: $\{y \mid 0 \leq y < \infty\}$

In Step 2, Professor K determines where the graph begins (zero) and that there is no ending along the y-axis. Professor K realizes that the lowest value of y is zero. Therefore, zero is the minimum value of y. Professor K also sees that positive infinity is the largest value of y. Therefore, ∞ is the maximum value of y. Professor K sets up an inequality statement to define the values of y ($0 \leq y < \infty$). Professor K states the Range as $\{y \mid 0 \leq y < \infty\}$.

Show Professor K that you understand how to determine the range and domain of relations by working the activities for this lesson.

Life Principle: *"The reputation of a thousand years may be determined by the conduct of one hour."*
—Japanese Proverb



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



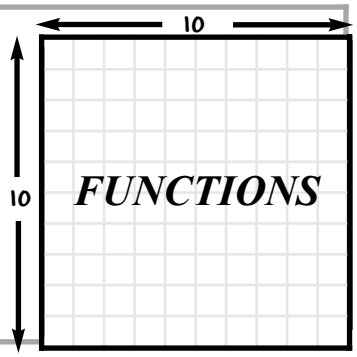
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Lesson 2

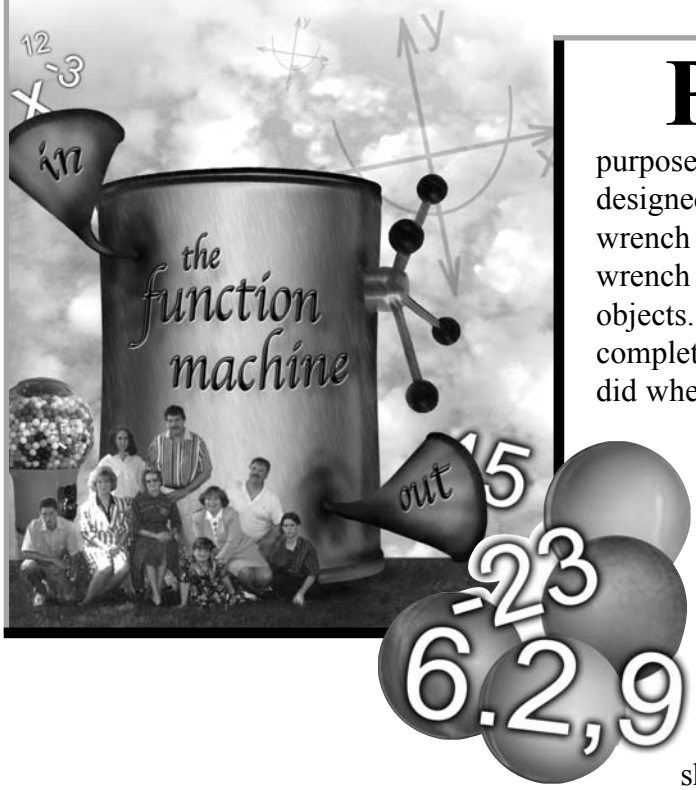
Graphs

Chapter 5



Section 1

Lessons 1-5



Professor K remembers that safety rules require a person to use tools for the purposes or ***function*** for which the tools were designed. Some people may use an adjustable wrench in place of a hammer. However, the wrench was not designed as a hammer to hit objects. Over time, the wrench will break completely, or at least not function as well as it did when new.

Mathematics has equations that are labeled as functions. A function in mathematics is like a machine into which you put one number and get out one and only one number. A function also draws a certain type of line or curve on a graph. In this lesson Professor K will introduce you to functions.

Look at **Figure 2.1** as Professor K shows you the graph of several functions.

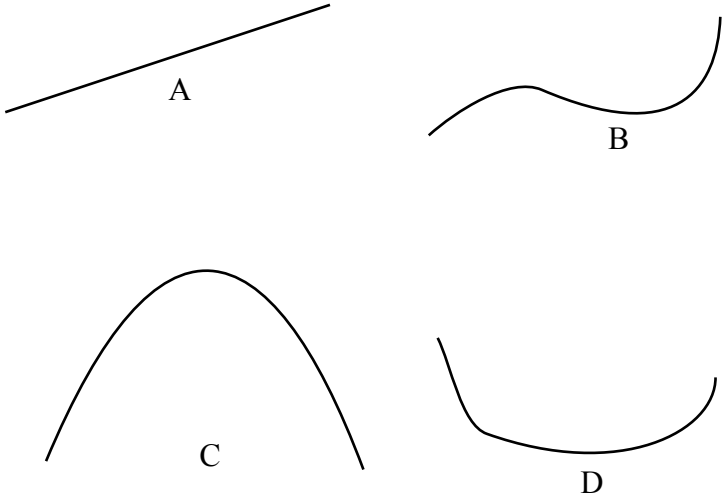


Figure 2.1

VOCABULARY

Function: a relation in which no two members of the domain are the same value
Vertical Line Test: test used to verify if a relation is a function
Ellipse: an oval shape; a flattened circle

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

All of the graphs in **Figure 2.1** are graphs of functions. Professor K wants you to notice that you can draw vertical lines through each graph, and each vertical line will cross the graph only once.

Look at **Figure 2.2**.

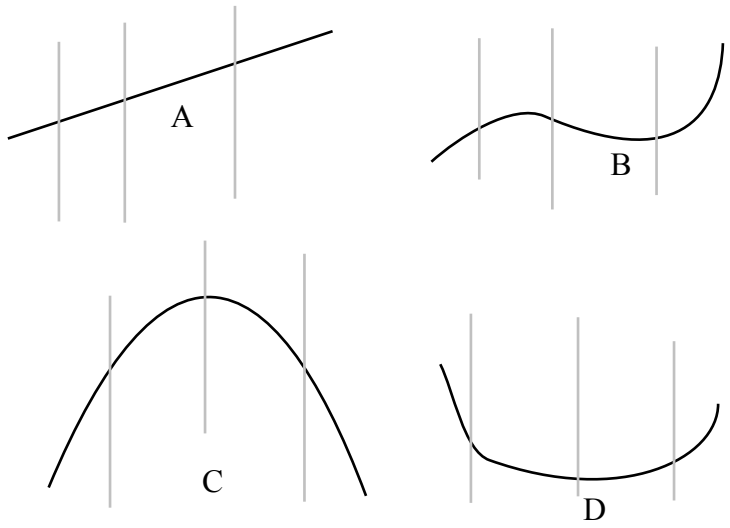
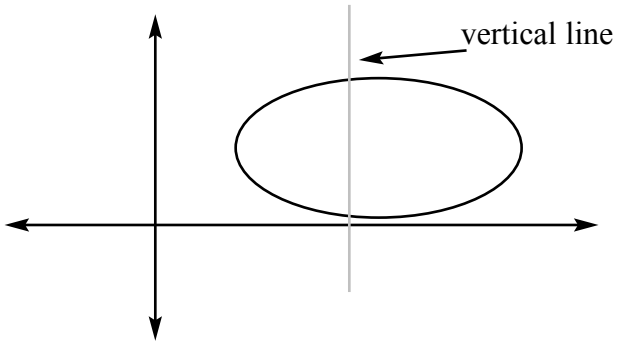


Figure 2.2

Professor K calls your attention to all the gray vertical lines in **Figure 2.2**. You could draw even more vertical lines and never intersect any of the graphs more than once with a vertical line. This is called the ***Vertical Line Test*** for determining from the graph of an equation whether an equation represents a function.

Look at **Example Set 1** while Professor K determines whether the graph presented represents a function.

Example Set 1

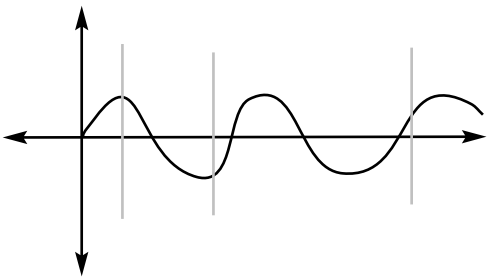


This drawing is the graph of an ***ellipse***. Using the vertical line test, Professor K sees that he can draw a vertical line that will intersect the graph *more* than once. Therefore, the equation of an ellipse is NOT a function.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Example Set 2



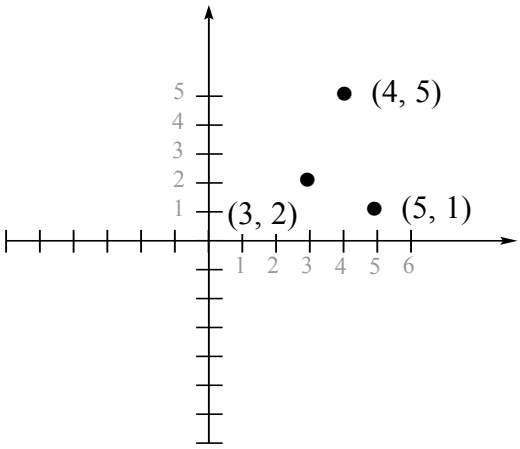
This illustration is the graph of a mathematical curve. Using the vertical line test, Professor K sees that he cannot draw a vertical line that will intersect the graph more than once. Therefore, the equation for this curve is a function.

When looking at a set of ordered pairs that are part of the solution to an equation, the abscissas of two or more points cannot be the same as for the equation to represent a function. Look at **Example Set 3** to see how Professor K determines if a set of ordered pairs represents a function.

Example Set 3

{(3, 2), (5, 1), (4, 5)}

The abscissas of the ordered pairs of this set are 3, 5, and 4. None of these numbers are the same, therefore, the set of ordered pairs represents a function.



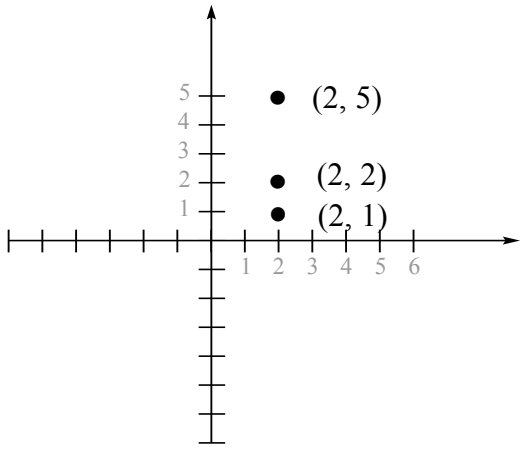
Notice that Professor K has plotted the points on the Cartesian coordinate system at the left. Notice that a vertical line through any one of the points would contain only that point and not any of the other points in the set. Put more simply, there are no two points directly above or below each other. Therefore, the set of points does represent a function.

Example Set 4

{(2, 2), (2, 1), (2, 5)}

The abscissas of the ordered pairs of this set are all 2. All of these numbers are the same, therefore, the set of ordered pairs does not represents a function.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Notice that Professor K has plotted the points on the Cartesian coordinate system at the left. Notice that a vertical line passes through at least two (in this case, all) of the points. Put more simply, there are two points directly above or below each other. Therefore, the set of points does not represent a function.

Show Professor K that you understand how to work problems using the Vertical Line Test and other properties of functions by working the activities for this lesson.

Life Principle



"All glory comes from
daring to begin."
—Anonymous

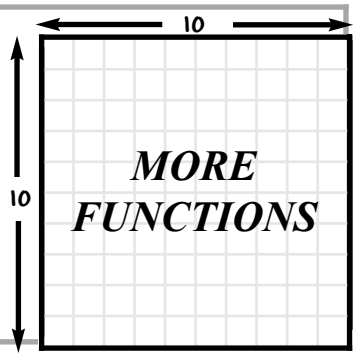
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Lesson 3

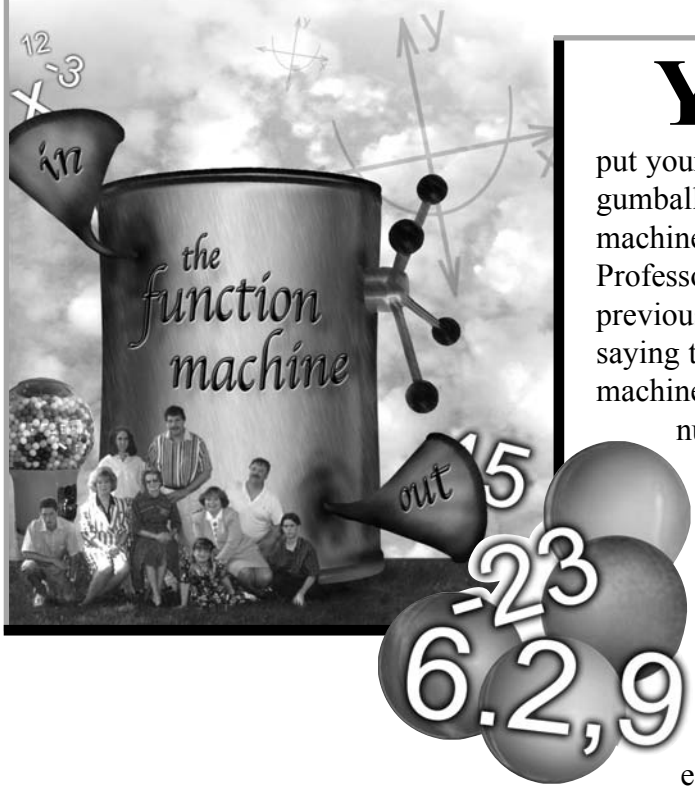
Graphs

Chapter 5



Section 1

Lessons 1-5





Professor K

You have probably put money into a gumball machine. You put your money into the machine and a gumball came out. The function of this machine was to ***dispense*** gumballs. Professor K reminds you that in the previous lesson he introduced you to functions by saying that a function is like a type of number machine. You put one number in and get another number out (such as money in, gumball out).

An ***equation*** containing two variables or letters is a number machine that produces a number when a value is substituted for one of the variables. In previous lessons, Professor K showed you how to determine patterns in a list of numbers. If you could write an expression for a term in the list as an equation, you would have a number machine into which you could put the number of the term and get out the value of that term. In a list of numbers (called *terms* for this lesson), each number in the list would be a separate *term*. With each term we have two things to consider: one being the *value* of the term, the other being the *number* of the term.

The *value* of the term is the actual number in the list. For instance, in a list of numbers: 5, 10, 20, 25..., the actual number is the value of the term. The first term in the list has a value of *5*, the second term has a value of *10*, etc. In the list of numbers: 40, 80, 120, 160..., the value of the first term is *40*, the value of the second term is *80*, the value of the fourth term is *160*, etc. The *value* of any number in a list is that number.

The *number* of the term is its position in the list of numbers. For example: first, second, third, fourth, fifth, or 60th. In the first list above, *20* is the fourth term in the list, *10*

VOCABULARY

Dispense: to release or give out; scatter
Equation: a statement of equality or inequality
t_n**:** an expression indicating the *n*th term in a list of numbers

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

is the second term and *5* is the first term. In the second list, *120* is the third term. The number of the term is found by starting with the first or smallest number in a list (the first term) and counting left to right or in ascending order to the term indicated. The counting number reached is the number of the term and is symbolized by "*n.*" The value of the *n*th term or any term in a list is *"t_n",* where *"t"* is the *term* and the *"n"* is the *number* of the term. To find the value of the term in the list of numbers, the list must be in ascending order. Ascending order means that the numbers must be in numerical order from the smallest number to the largest number. The *"n*th*"* term is any term in the list of numbers. If the expression we write to find the value of the *"n*th*"* term (t_n) is a correct expression, we will be able to find the value for any term (t_n) in the list. Professor K shows you this in **Example Set 1**.

Example Set 1

Determine an expression for the *n*th term in the list of numbers below:
0, 1, 2, 3, 4, 5, …

Step 1

first term, $n = 1$ $t_n = 0$
second term, $n = 2$ $t_n = 1$
third term, $n = 3$ $t_n = 2$
fourth term, $n = 4$ $t_n = 3$

In Step 1, Professor K sets up the terms (n) and their values (t_n), to establish a pattern. Professor K points out that the values of the terms are increasing by one, and that the value of the term is one less than the number of the term. Therefore, Professor K uses $n - 1$ as the expression for the *n*th term of the list of numbers.

Step 2

$t_n = n - 1$

In Step 2, Professor K writes an equation using the expression for the *n*th term (t_n) of this list of numbers. This equation is the number machine that you can use to obtain the *n*th term of the list. (Term number (n) goes in; value of term (t_n)comes out.)

Look at **Example Sets 2** and **3** to see how Professor K uses similar expressions to $t_n = n - 1$, $y = x + 5$ in Example Set 2 and $y = x - 8$ in Example Set 3, to determines the solution to equations that represent functions. Professor K then shows you how to write the answers. Review Chapter 1 Lesson 12 for more information on developing this type of expression.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Example Set 2

Determine the solutions to the function below for the values of x between 0 and 10. Write the solutions as ordered pairs.

$y = x + 5$

Step 1

$0 < x < 10$
$x = 1$: $y = 1 + 5$; $y = 6$
$x = 2$: $y = 2 + 5$; $y = 7$
$x = 3$: $y = 3 + 5$; $y = 8$
$x = 4$: $y = 4 + 5$; $y = 9$
$x = 5$: $y = 5 + 5$; $y = 10$
$x = 6$: $y = 6 + 5$; $y = 11$
$x = 7$: $y = 7 + 5$; $y = 12$
$x = 8$: $y = 8 + 5$; $y = 13$
$x = 9$: $y = 9 + 5$; $y = 14$

In Step 1, Professor K determines that the values of $x = 1, 2, 3, 4, 5, 6, 7, 8$ and 9. Professor K then substitutes those values of x into the equation and performs the calculations to obtain a corresponding value of y for each value of x.

Step 2

{(1, 6), (2, 7), (3, 8),
(4, 9), (5, 10), (6, 11)
(7, 12), (8, 13), (9, 14)}

In Step 2, Professor K writes the solutions as ordered pairs.

Example Set 3

Determine the solutions to the function below for the values of x between and including -2 and 5. Write the solutions as ordered pairs.

$y = x - 8$

Step 1

$-2 \leq x \leq 5$
$x = -2$: $y = -2 - 8$; $y = -10$
$x = -1$: $y = -1 - 8$; $y = -9$
$x = 0$: $y = 0 - 8$; $y = -8$
$x = 1$: $y = 1 - 8$; $y = -7$
$x = 2$: $y = 2 - 8$; $y = -6$
$x = 3$: $y = 3 - 8$; $y = -5$
$x = 4$: $y = 4 - 8$; $y = -4$
$x = 5$: $y = 5 - 8$; $y = -3$

In Step 1, Professor K determines that the values of $x = -2, -1, 0, 1, 2, 3, 4$ and 5. Professor K then substitutes those values of x into the equation and performs the calculations to obtain a corresponding value of y for each value of x.

Step 2

{(-2, -10), (-1, -9), (0, -8),
(1, -7), (2, -6), (3, -5),
(4, -4), (5, -3)}

In Step 2, Professor K writes the solutions as ordered pairs.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Show Professor K that you understand how to work problems using the properties you have learned about functions by working the activities for this lesson.

LIFE PRINCIPLE



"When eating fruit, think of the person who planted the tree."

—Vietnamese proverb

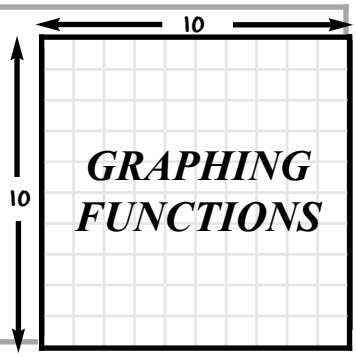
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Lesson 4

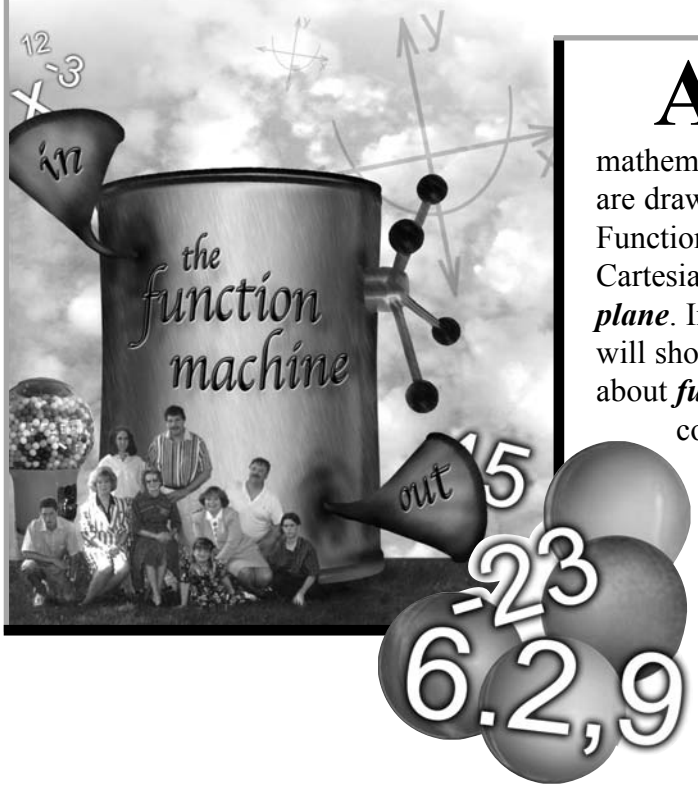
Graphs

Chapter 5



Section 1

Lessons 1-5





A picture is said to be worth a thousand words. When mathematicians draw a graph they are drawing a picture of an equation. Functions are usually graphed on a Cartesian coordinate system or ***plane***. In this lesson, Professor K will show you how to take what you have learned about ***functions*** and ***graph*** them on a Cartesian coordinate plane.

Follow along with Professor K as he shows you how to graph a function after finding some of the solutions to the equation.

Example Set 1

Graph the function, $y = x + 2$ on a Cartesian coordinate plane.

Step 1 $x = 0$: $y = 0 + 2$; $y = 2$
$x = 1$: $y = 1 + 1$; $y = 3$
$x = 2$: $y = 2 + 2$; $y = 4$

In Step 1, Professor K chooses three numbers at random as values for x. Professor K chooses $x = 0$, 1, and 2. Professor K substitutes the values that he has chosen for x into the equation $y = x + 2$ and calculates the value of y for each value of x.

Step 2 {(0, 2), (1, 3), (2, 4)}

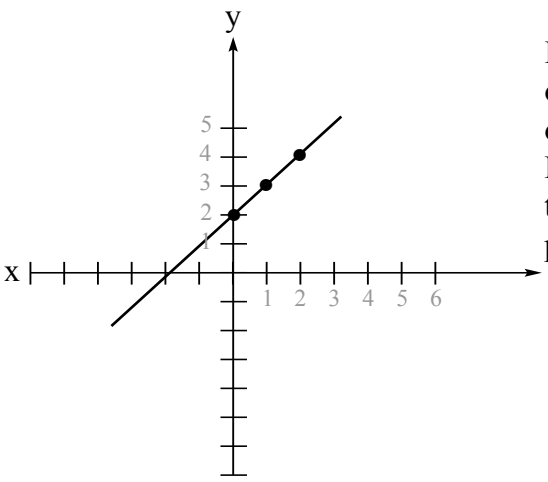
In Step 2, Professor K places the values that he determined in Step 1 into ordered pairs as shown.

VOCABULARY

Plane: in geometry, a two dimensional area
Function: a relation or equation in which no two members of the domain are the same
Graph: to draw a figure on a coordinate system

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Step 3



In Step 3, Professor K plots these points on the graph. Professor K connects two of the points with a line (2, 4) and (1, 3). Professor K extends the line downward to the left and notices that the line also passes through the third point, (0, 2).

As seen in **Example Set 1**, Professor K knew that it took only two points to draw a line. He plotted a third point to determine that he had drawn the line correctly. Professor K did draw the line correctly because the line also passed through the point (0, 2). Every solution that Professor K finds for this equation should be contained in the same line. Notice also from the graph, you can see that $y = x + 2$ is a function by using the Vertical Line Test.

Look at **Examples Set 2** as Professor K graphs another function for you.

Example Set 2

Graph the function, $y = 2x + 1$ on a Cartesian coordinate plane.

Step 1

$x = 0$: $y = 0 + 1$; $y = 1$
$x = 1$: $y = 2 + 1$; $y = 3$
$x = -1$: $y = -2 + 1$; $y = -1$

In Step 1, Professor K chooses three numbers at random as values for x. Professor K chooses $x = 0$, 1, and -1. Professor K substitutes the values that he has chosen for x into the equation $y = 2x + 1$ and calculates the value of y for each value of x.

Step 2

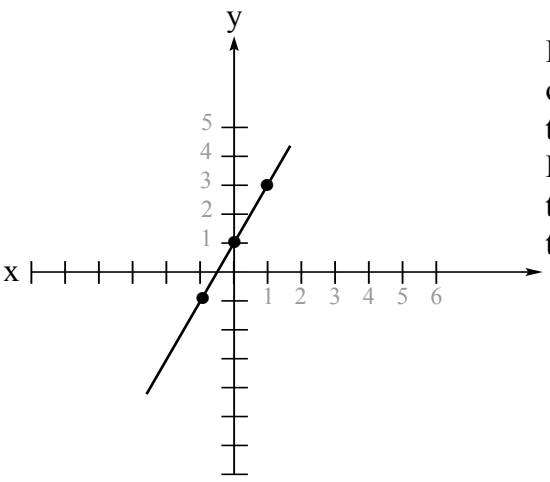
{(0, 1), (1, 3), (-1, -1)}

In Step 2, Professor K places the values that he determined in Step 1 into ordered pairs as shown.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Step 3



In Step 3, Professor K plots these points on the graph. Professor K connects two of the points with a line (1, 3) and (0, 1). Professor K extends the line downward to the left and notices that the line also passes through the third point, (-1, -1).

In **Example Set 2**, Professor K again plotted a third point to determine that he had drawn the line correctly. Professor K did draw the line correctly because the line also passed through the point (-1, -1). Every solution that Professor K finds for this equation should be contained in the same line. Notice also from the graph, you can see that $y = 2x + 1$ is a function by using the Vertical Line Test.

Show Professor K that you understand how to graph functions by working the activities for this lesson.

LIFE PRINCIPLE



"The best mirror is a friend's eye."

—Gaelic Proverb

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

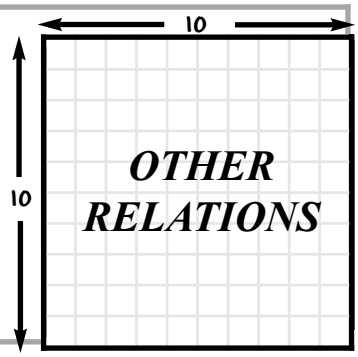


©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Geometry

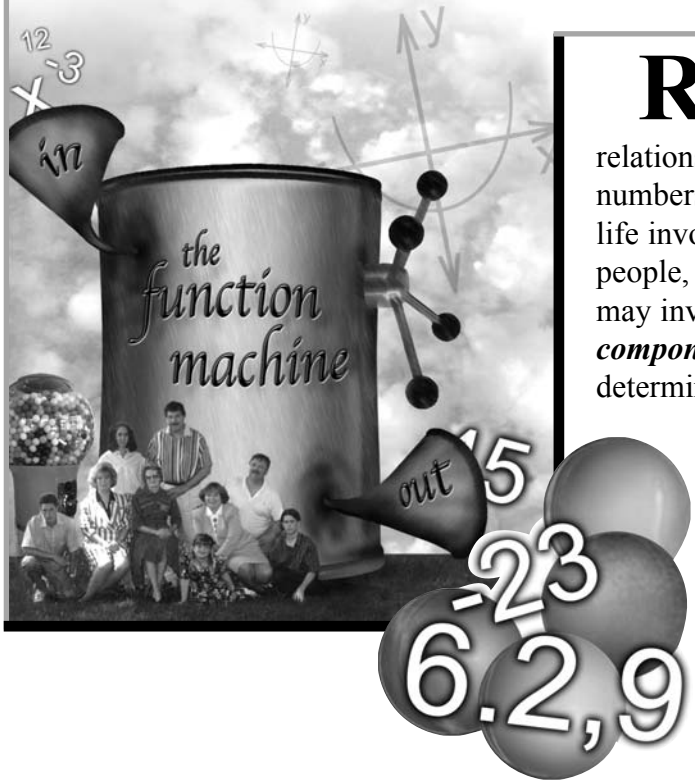
Lesson 5

Chapter 5



Section 1

Lessons 1-5





Professor K

Remember, in earlier lessons Professor K described a relationship between two variables or numbers. Just as relationships in your life involve more than just two people, relationships in mathematics may involve three or more ***components***. Consider the formula for determining the volume of a rectangular prism,

$$V = l \times w \times h.$$

A relationship of these lengths can be shown using what is called an ***ordered triplet*** instead of an ordered pair. An ordered pair was used when an equation contained two variables. An ordered triplet is used when an equation contains three variables, as does the formula for the volume of a rectangular prism.

Follow along with Professor K in the examples as he shows you how to work equations containing three variables and how to write the solution in ordered triplets.

VOCABULARY

Components: parts of something
Ordered triplet: method of expressing within parentheses, the solution to an equation containing three variables; similar to an ordered pair

Example Set 1

If the volume of a rectangular prism is 100 cubic feet, determine the possible dimensions of the prism. Write the solution in the form of an ordered triplet.

Step 1

$V = l \times w \times h$
$100 = l \times w \times h$

In Step 1, Professor K substitutes the volume given in the problem into the formula for the volume of a rectangular prism.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Step 2	$l = 2$ $100 = 2 \times w \times h$ $50 = w(h)$	In Step 2, Professor K chooses a value for the length, l. Professor K chooses 2 as that value and places the value in the formula from Step 1. Professor K then simplifies the equation by dividing by two as shown.
Step 3	$w = 2$ $50 = 2(h)$ $h = 25$	In Step 3, Professor K chooses a value for the width, w. Professor K chooses 2 as that value and places the value in the equation, $50 = w(h)$ from Step 2. Professor K then simplifies the equation by dividing by two as shown.
Step 4	(l, w, h): (2, 2, 25)	In Step 4, Professor K now places the solutions to the three variables into an ordered triplet.

Example Set 2

Determine if the ordered triplet: 4, 5, 10 (for l, w, h) is a solution to $200 = l \times w \times h$.

Step 1	$l = 4$ $200 = 4 \times w \times h$ $50 = w(h)$	In Step 1, Professor K substitutes the first value, $l = 4$ of the ordered triplet into the equation given. Professor K then simplifies the equations by dividing both sides by 4 to obtain the equation, $50 = w(h)$.
Step 2	$w = 5$ $50 = 5(h)$ $10 = h$	In Step 2, Professor K places the value of the next coordinate of the ordered triplet into the equation from Step 1 and then divides both sides by 5 to obtain $h = 10$.

By placing each coordinate into the equation and simplifying, Professor K determined that the ordered triplet, (4, 5, 10) is a solution to the equation $200 = l \times w \times h$.

Show Professor K that you understand how to work problems involving three variables and ordered triplets by working the activities for this lesson.

Life Principle



"You may find the worst enemy or best friend in yourself."

—English Proverb

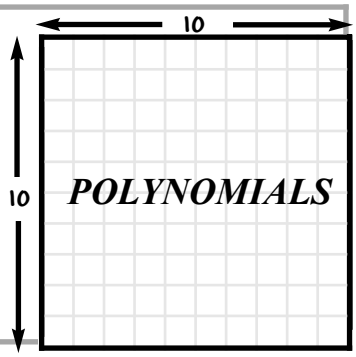
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Algebra

Lesson 6

Chapter 5



Section 2

Lessons 6-10





When doing business in the world today, people sign contracts. Contracts are agreements between two individuals or groups for one to supply some goods or services to the other for a specified price. You have probably heard of professional athletes signing contracts to play for particular teams. Such contracts consist of many ***terms*** and conditions that spell out what the players are expected to do to receive the amount of salaries listed in the contracts.



Mathematics has expressions with many terms or parts. ***Polynomial*** is a word used to describe something that has many (poly) names (nomial). In mathematics, an algebraic expression of two or more terms separated by addition or subtraction symbols is called a *polynomial*. In this lesson, Professor K will show you various polynomials and how to identify them.

Certain polynomials have special names that describe how many terms are contained in the polynomial.

1. A ***monomial*** contains one term, such as $3x$, $24y$, or $5xyz$.
2. A ***binomial*** contains two (bi) terms separated by addition or subtraction symbols, such as $x + 2$ or $x + y$.
3. A ***trinomial*** contains three (tri) terms separated by addition or subtraction symbols, such as:

   $x^2 + x + 2$, $x + y + z$ or $x + y - 2$.

Polynomials containing four or more terms are named as follows:

4. A polynomial of four terms contains four terms separated by addition or subtraction symbols:

   $x^3 + x^2 + x + 2$; $x + y + z + 5$

VOCABULARY

Term: parts of an algebraic expression separated by addition or subtraction
Polynomial: an algebraic expression of more than one or many terms
Monomial: a mathematical expression containing one term
Binomial: a polynomial containing two terms
Trinomial: a polynomial containing three terms

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

5. A polynomial of five terms contains five terms separated by addition or subtraction symbols:

$$2a + 3b - 5c + d - f; \qquad x^4 + 10x^3 + x^2 + 5x + 2$$

6. A polynomial of six terms contains six terms separated by addition or subtraction symbols:

$$2a + 3b - 5c + d - 3e + f, \qquad 4x^5 - 2x^4 + x^3 + x^2 + x + 2$$

For higher numbered polynomials, simply place the number of terms in the blank of the words, "polynomial of _____ terms." Look at **Example Set 1** as Professor K identifies some polynomials.

Example Set 1

a) $23x$	Professor K determines this to be a monomial because the expression has only one term that contains multiplication, $23 \cdot x$.
b) $x + 8$	Professor K determines this polynomial to be a binomial because the expression has two terms separated by addition.
c) $3x - 4y + 25z$	Professor K determines this polynomial to be a trinomial because it has three terms separated by addition and subtraction.
d) $a + 2b + 3c - 4d + 5e - 9f + 11g - 45$	Professor K determines this polynomial to be a polynomial of eight terms because it has eight terms separated by addition and subtraction.

Look at **Example Set 2** as Professor K identifies more polynomials.

Example Set 2

a) x	Professor K determines this to be a monomial because the expression has only one term that contains multiplication, $1 \cdot x$.
b) $5x + 4$	Professor K determines this polynomial to be a binomial because the expression has two terms, $5x$ and 4 separated by addition.
c) $x + 8y + 3z$	Professor K determines this polynomial to be a trinomial because it has three terms, x, $8y$ and $3z$ separated by addition.
d) $7a - b + 10c + 2d - 6e + 14f + 21g + 7h + 8$	Professor K determines this polynomial to be a polynomial of nine terms because it has nine terms separated by addition and subtraction.

Show Professor K that you understand how to identify polynomials by working the activities for this lesson.

LIFE PRINCIPLE



"From the little spark may burst a mighty flame."

—Dante Alighieri
(1265-1321)

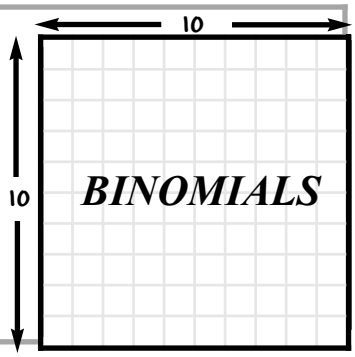
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Algebra

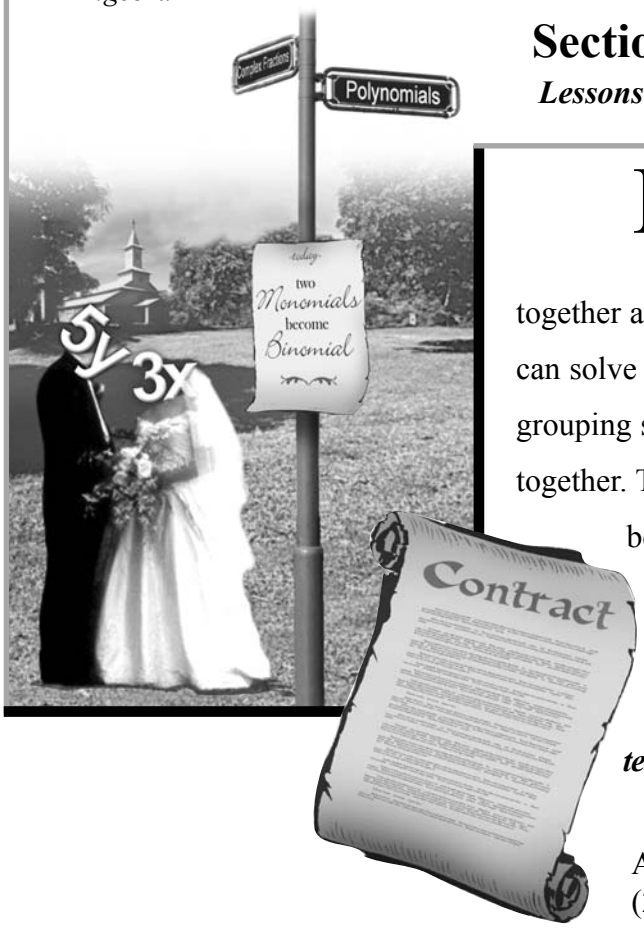
Lesson 7

Chapter 5



Section 2

Lessons 6-10





Many problems you face each day often seem to add together and overwhelm you. However, you can solve these everyday problems by grouping similar or like portions of the problems together. This process helps you to manage your time better for solving the entire problem.

Two or more polynomials can be added together. Look at **Example Set 1** to see how Professor K adds two ***binomials*** by grouping ***like terms*** together to obtain a simplified solution.

Example Set 1

Add the following binomials.
$(2x + 4) + (3x + 9)$

VOCABULARY

Binomial: a polynomial containing two terms
Like terms: terms containing the same variable (letters) as factors

Step 1 $2x + 4 + 3x + 9 = 2x + 3x + 4 + 9$

Professor K begins to determine the sum of the binomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $3 + 1 = 1 + 3$.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Step 2	$(2 + 3)x + (4 + 9) = 5x + 13$	In Step 2, Professor K combines like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $5x + 13$ is the binomial sum of $(2x + 4)$ and $(3x + 9)$.

Example Sets 2 and 3 show how Professor K adds more binomials together.

Example Set 2

Add the following binomials.

$(x + 2) + (x + 2)$

Step 1	$x + 2 + x + 2 = x + x + 2 + 2$	Professor K begins to determine the sum of the binomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $3 + 1 = 1 + 3$.
Step 2	$(1 + 1)x + (2 + 2) = 2x + 4$	In Step 2, Professor K combines like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $2x + 4$ is the binomial sum of $(x + 2)$ and $(x + 2)$.

Example Set 3

Add the following binomials.

$(4x + 8) + (x + 7)$

Step 1	$4x + 8 + x + 7 = 4x + x + 8 + 7$	Professor K begins to determine the sum of the binomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $3 + 1 = 1 + 3$.
Step 2	$(4 + 1)x + (8 + 7) = 5x + 15$	In Step 2, Professor K combines like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $5x + 15$ is the binomial sum of $(4x + 8)$ and $(x + 7)$.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

You can add various other polynomials by following the same steps as in these examples. Show Professor K that you understand how to add binomials by working the activities for this lesson.

LIFE PRINCIPLE



"A good example is like a bell that calls many to church."

—Danish Proverb

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



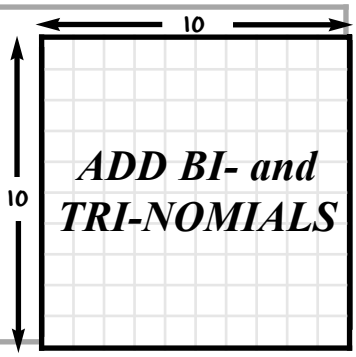
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Algebra

Lesson 8

Chapter 5



Section 2

Lessons 6-10





Those problems you face each day can increase from one or two components to many, as explained in the previous lesson. You want to deal with one aspect of the problem at a time; but sometimes, several parts seem to be challenging you all at once, adding to your ***dilemma***.

Complex problems are like polynomials: they can be solved one step at a time. In this lesson, Professor K will add two more polynomials together. Look at **Example Set 1** to see how Professor K adds binomials and trinomials.

Example Set 1 shows how Professor K adds a binomial to a trinomial.

Example Set 1

Add the following polynomials.
$(5x - 6) + (6x - y + 8)$

Step 1 $5x - 6 + 6x - y + 8 = 5x + 6x + 8 - 6 - y$

Professor K begins to ***determine*** the sum of the binomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $3 + 1 = 1 + 3$ or $a + b = b + a$.

VOCABULARY

Dilemma: a perplexing or confusing situation
Determine: to figure out a situation or find a solution

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Step 2	$(5 + 6)x + (8 - 6) - y =$ $11x + 2 - y$	In Step 2, Professor K combines like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $11x + 2 - y$ is the trinomial sum of $(5x - 6)$ and $(6x - y + 8)$.

In **Example Set 2**, Professor K shows how to add a binomial to a trinomial, again.

Example Set 2

Add the following polynomials.

$(x + 5) + (3x - y + 9)$

Step 1	$x + 5 + 3x - y + 9 =$ $x + 3x + 5 + 9 - y$	Professor K begins to determine the sum of the binomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $a + b = b + a$.
Step 2	$(1 + 3)x + (5 + 9) - y =$ $4x + 14 - y$	In Step 2, Professor K combines like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $4x + 14 - y$ is the trinomial sum of $(x + 5)$ and $(3x - y + 14)$.

Once again in **Example Set 3**, Professor K shows how to add a binomial to a trinomial.

Example Set 3

Add the following polynomials.

$(5x - 1) + (x + 2y - 4)$

Step 1	$5x - 1 + x + 2y - 4 =$ $5x + x - 1 - 4 + 2y$	Professor K begins to determine the sum of the binomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $a + b = b + a$.
Step 2	$(5 + 1)x + (-1 - 4) + 2y =$ $6x - 5 + 2y$	In Step 2, Professor K combines like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $6x - 5 + 2y$ is the trinomial sum of $(5x - 1)$ and $(x + 2y - 4)$.

You can add various other polynomials by following the same steps as in these examples. Show Professor K that you understand how to add binomials and trinomials by working the activities for this lesson.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

LIFE PRINCIPLE



"He who is caught in a lie is not believed when he tells the truth."

—Spanish Proverb

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



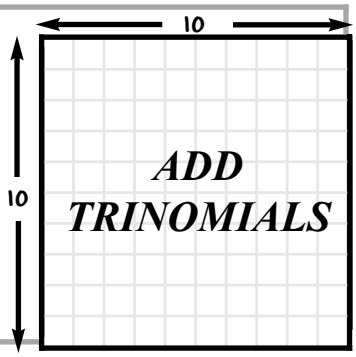
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Algebra

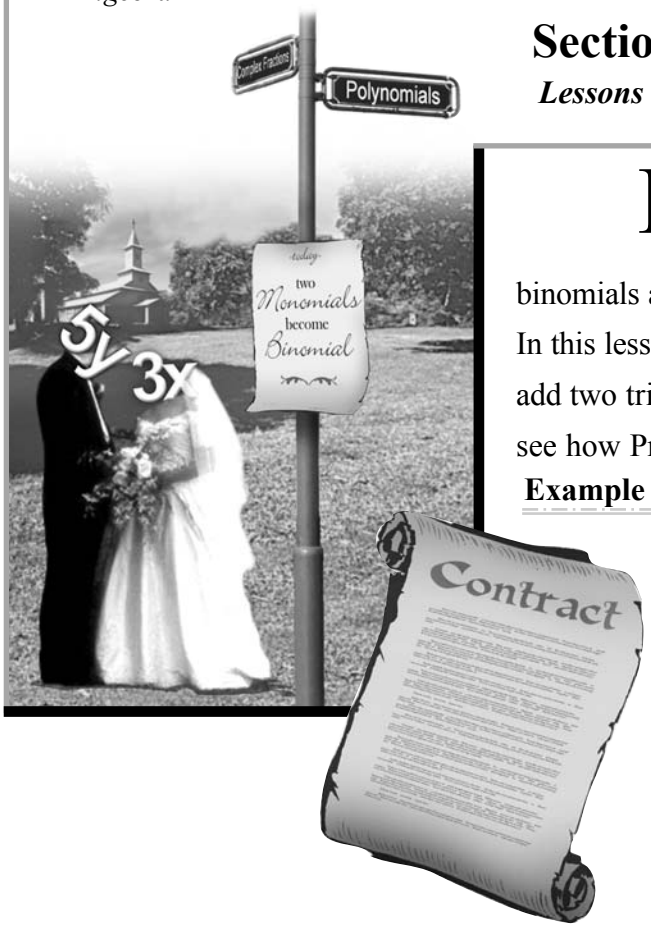
Lesson 9

Chapter 5



Section 2

Lessons 6-10





In previous lessons, Professor K showed you how to add two binomials and to add binomials to trinomials. In this lesson Professor K will show you how to add two trinomials together. Look at **Example Set 1** to see how Professor K adds two trinomials.

Example Set 1

Add the following polynomials.
$(3a - 16b + 4c) + (2a - 2b - 4c)$

Step 1

$3a - 16b + 4c + 2a - 2b - 4c =$
$3a + 2a - 16b - 2b + 4c - 4c$

Professor K begins to determine the sum of the trinomials by ***dropping*** the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $a + b = b + a$.

Step 2

$(3 + 2)a + (- 16 - 2)b + (4 - 4)c =$
$5a - 18b + 0c = 5a - 18b$

In Step 2, Professor K ***combines*** like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $5a - 18b$ is the binomial sum of $(3a - 16b + 4c)$ and $(2a - 2b - 4c)$.

VOCABULARY

Dropping: removing; taking away
Combine: put together as one

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

In **Example Set 2**, Professor K shows you how to add two other trinomials.

Example Set 2

Add the following polynomials.

$(4x - 6y + 4) + (6x - 2y + 7)$

Step 1 $4x - 6y + 4 + 6x - 2y + 7 = 4x + 6x - 6y - 2y + 4 + 7$

Professor K begins to determine the sum of the trinomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $a + b = b + a$.

Step 2 $(4 + 6)x + (- 6 - 2)y + (4 + 7) = 10x - 8y + 11$

In Step 2, Professor K combines like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $10x - 8y + 11$ is the trinomial sum of $(4x - 6y + 4)$ and $(6x - 2y + 7)$.

In **Example Set 3**, Professor K shows you how to add two more trinomials.

Example Set 3

Add the following polynomials.

$(5x + 2y + 3z) + (2x - 2y - 2z)$

Step 1 $5x + 2y + 3z + 2x - 2y - 2z = 5x + 2x + 2y - 2y + 3z - 2z$

Professor K begins to determine the sum of the trinomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $a + b = b + a$.

Step 2 $(5 + 2)x + (2 - 2)y + (3 - 2)z = 7x + 0y + z = 7x + z$

In Step 2, Professor K combines like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $7x + z$ is the binomial sum of $(5x + 2y + 3z)$ and $(2x - 2y - 2z)$.

You can add various other polynomials by following the same steps as in these examples. Show Professor K that you understand how to add trinomials by working the activities for this lesson.

Life Principle



"Patience is the companion of wisdom."

—St. Augustine
(A.D. 354-430)

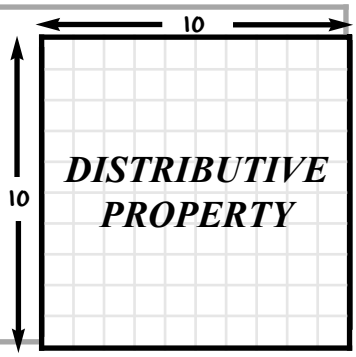
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Lesson 10

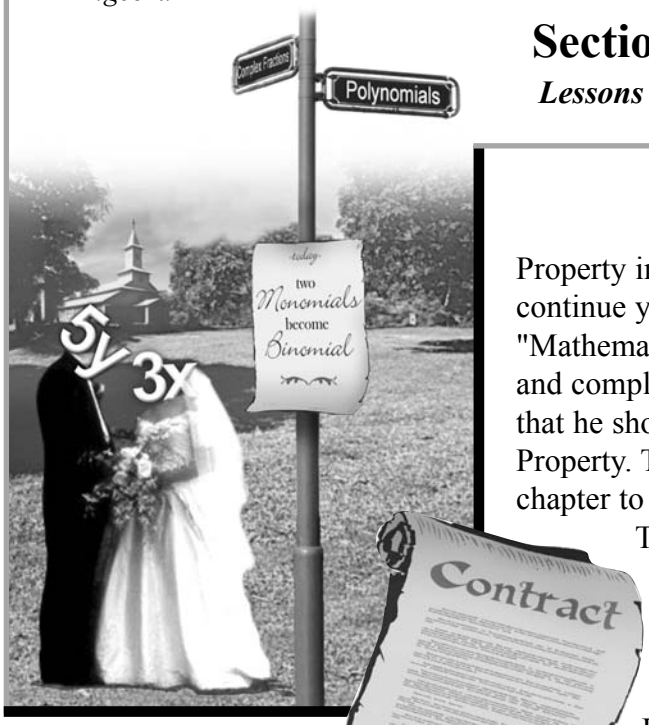
Algebra

Chapter 5



Section 2

Lessons 6-10





Professor K showed you how to use the Distributive Property in previous lessons. Before you continue your journey through this part of "Mathematics Land" known as polynomials and complex fractions, Professor K believes that he should present a review of the Distributive Property. The Distributive Property will be used in this chapter to simplify various mathematical expressions.

The Distributive Property is an important mathematical property.

Often, algebraic expressions contain grouping symbols that separate numbers and variables in multiplication. In this lesson Professor K will show you how to simplify algebraic expressions using the Distributive Property. Look at the examples to see how Professor K simplifies such expressions. In **Example Set 1** Professor K shows you how the Distributive Property can be ***utilized*** in numerical expressions.

Example Set 1

Simplify the following expressions.

a) $4(3 + 2)$

$4 \cdot 3 + 4 \cdot 2$

$12 + 8 = 20$

Professor K multiplies the 4 times each number within the parentheses. Professor K thinks of this as distributing the 4 within the parentheses. Professor K performs the multiplications and adds the products to obtain the solution, 20. (This is the same solution that you would obtain if you used PEMDAS. $3 + 2 = 5$; $4(3 + 2) = 4(5) = 20$.) This is an example of the Distributive Property across addition.

VOCABULARY

Utilized: to put to use for a specific purpose

Viable: continually effective

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

b) $2(7 - 5)$
$2 \bullet 7 - 2 \bullet 5$
$14 - 10 = 4$

Professor K multiplies the 2 times each number within the parentheses. Think of this as distributing the 2 within the parentheses. Professor K performs the multiplications then subtracts the products to obtain the solution, 4. (This is the same solution that you would obtain if you used PEMDAS. $7 - 5 = 2$; $2(7 - 5) = 2(2) = 4$.) This is an example of the Distributive Property across subtraction.

Notice that in numerical expressions like those in **Example Set 1**, the simplest way to obtain a solution is to use PEMDAS. Professor K simplified these expressions to demonstrate how the Distributive Property works and so you could see that the Distributive Property is a ***viable*** mathematical method of simplification. Look at **Example Set 2** as Professor K shows you an example of how to simplify an algebraic expression (an expression containing variables and grouping symbols), using the Distributive Property.

Example Set 2

Simplify the following algebraic expressions.

a) $4(x + 2) =$
$4 \bullet x + 4 \bullet 2 =$
$4x + 8$

Professor K simplifies the algebraic expression of $4(x + 2)$ by using the Distributive Property across addition as he did in **Example Set 1**. Professor K multiplies the 4 times each of the two terms within the parentheses, separated by an addition symbol. Professor determines the solution to be $4x + 8$. Professor K cannot use PEMDAS because variables and numbers cannot be combined.

b) $2(x - 5) =$
$2 \bullet x - 2 \bullet 5 =$
$2x - 10$

Professor K simplifies the algebraic expression of $2(x - 5)$ by using the Distributive Property across subtraction as he did in **Example Set 1**. Professor K multiplies the 2 times each of the two terms within the parentheses, separated by a subtraction symbol. Professor K determines the solution to be $2x - 10$. Again, Professor K cannot use PEMDAS because variables and numbers cannot be combined

---

Show Professor K that you understand the Distributive Property by working the activities for this lesson.

LIFE PRINCIPLE



"I am convinced that life is 10 percent what happens to me and 90 percent how I react to it."

—Charles Swindoll

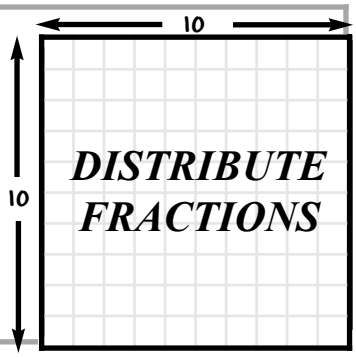
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Lesson 11

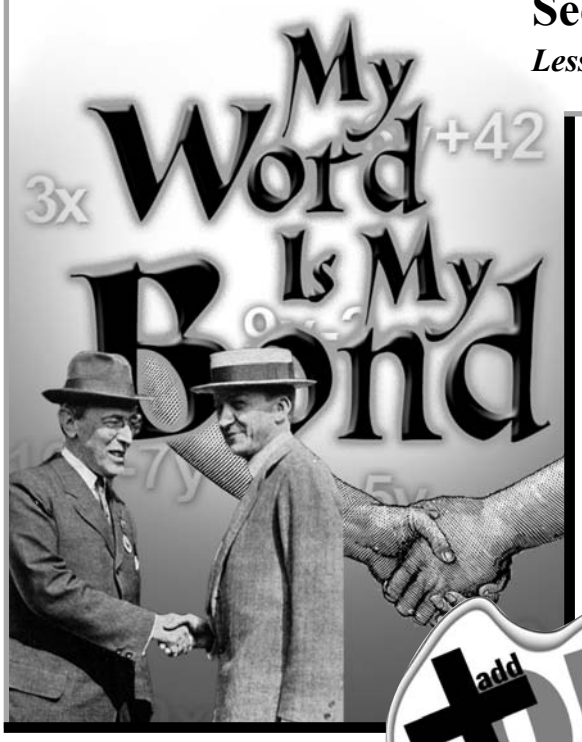
Algebra

Chapter 5



Section 3

Lessons 11-15







Professor K reviewed the Distributive Property in the previous lesson. In this lesson Professor K will ***extend*** that review to include using the Distributive Property to simplify fractions. Study this process well, as you will use it later in this chapter to simplify various ***fractional*** expressions. In **Example Set 1**, Professor K shows you how the Distributive Property is used to simplify fractional expressions.

Example Set 1 shows how Professor K simplifies a fraction that contains an algebraic expression.

Example Set 1

Simplify the following fraction.

$$\frac{3(x+3)}{5(x-1)}$$

Step 1 $\dfrac{3 \bullet x + 3 \bullet 3}{5 \bullet x - 5 \bullet 1}$

Professor K uses the Distributive Property to simplify the algebraic expression. Professor K performs the multiplications according to the Distributive Property in the numerator and then the denominator to obtain the solution.

Step 2 $\dfrac{3x+9}{5x-5}$

VOCABULARY

Extend: take to a new level; put to a greater use
Fractional: a part of; having to do with fractions

In **Example Set 2**, Professor K shows you how to simplify an expression containing multiple grouping symbols and variables.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Example Set 2

Simplify the following expression.

$$\frac{2[3(x+1)-2]}{3[x(x-1)+x]}$$

For ease of explanation, Professor K will simplify the numerator first and then the denominator. Follow his explanation.

Step 1

$2[3(x + 1) - 2]$
$2[3 \bullet x + 3 \bullet 1 - 2]$
$2[3x + 3 - 2]$
$2[3x + 1]$
$2 \bullet 3x + 2 \bullet 1$
$6x + 2$

In Step 1, Professor K simplifies the numerator using PEMDAS and the Distributive Property. Professor K looks within the square brackets and distributes the three within the parentheses. He then combines like terms within the square brackets. Professor K's final step is to distribute the two into the square brackets to obtain the solution.

Step 2

$3[x(x - 1) + x]$
$3[x \bullet x - x \bullet 1 + x]$
$3[x^2 - x + x]$
$3[x^2]$
$3 \bullet x^2$
$3x^2$

In Step 2, Professor K simplifies the denominator using PEMDAS and the Distributive Property. Professor K looks within the square brackets and distributes the x within the parentheses. He then combines like terms within the square brackets. Professor K's final step is to distribute the three into the square brackets to obtain the solution.

Step 3

$$\frac{6x+2}{3x^2}$$

In Step 3, Professor K combines Steps 1 and 2 and places the answers in the numerator and denominator to obtain the final solution.

In Step 2, Professor K determined that $x \bullet x = x^1 \bullet x^1 = x^{1+1} = x^2$. Professor K added the exponents. Professor K further determined that $-x + x = -1x + 1x = (-1 + 1)x = 0x = 0$. Professor K added the numbers in front of the x (the coefficients of x). Remember, a variable alone is understood to have a coefficient of one.

Show Professor K that you understand how to simplify fractions containing variables and grouping symbols that require using the Distributive Property by working the activities for this lesson.

Life Principle



"It's never too late–in fiction or in life–to revise."

—Nancy Thayer, Writer

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

x+3=7

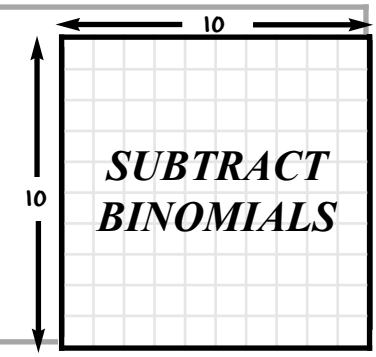
Lesson 12

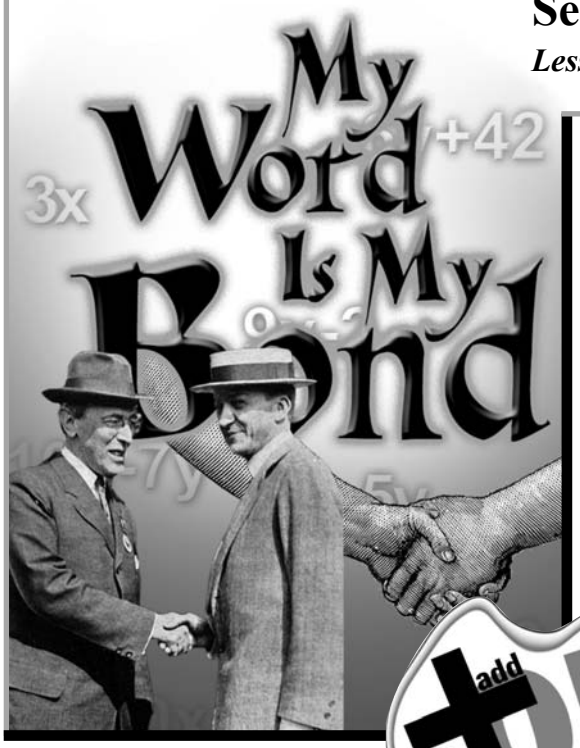
Algebra

Chapter 5

Section 3

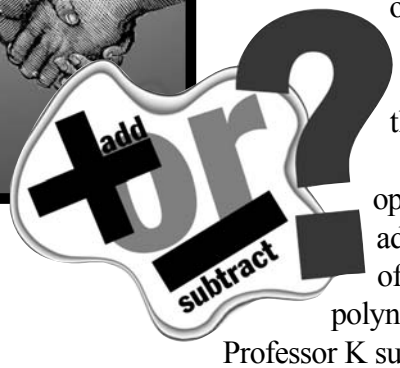
Lessons 11-15







Many times people do not do the right thing, but instead they do the opposite of what they promise. Signing a ***contract*** is the same as making a promise. In the past, men had only to give their word on something and that was all that was needed to see that it was done. Today, however, men often attempt to do the opposite of what was agreed upon because the agreement was not in writing or legally binding. Ancient Hebrew writings proclaimed that a man should not make a ***vow*** (promise) that he would not keep.



You know that subtraction is the opposite of addition. Not only can you add polynomials as you did in Lesson 1 of this chapter, but you can also subtract polynomials. Look at **Example Set 1** as Professor K subtracts two binomials.

Example Set 1

Subtract the following binomials.

$(8x + 4) - (3x + 2)$

Step 1 $8x + 4 - 3x - 2 =$
$8x - 3x + 4 - 2$

Professor K begins to determine the difference of the binomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $a + b = b + a$.

VOCABULARY

Contract: an agreement between two or more parties, usually in writing
Vow: a solemn promise
Coefficient: the multiplier of a variable; the number in front of a variable

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Step 1a	$-(3x + 2) = -1(3x + 2) =$ $-1 \bullet 3x + (-1) \bullet 2 =$ $-3x - 2$	In Step 1a, Professor K shows you how he used the Distributive Property to distribute the negative sign within the parentheses of the expression, $-(3x + 2)$. A -1 is understood to be in front of the parentheses. Therefore, Professor K distributes the -1 into $3x + 2$ by multiplying each term times -1. This changes all the signs to obtain $-3x - 2$. Professor K used this answer in Step 1 for the rearrangement of the terms to complete the necessary operation.
Step 2	$8x - 3x + 4 - 2 =$ $(8 - 3)x + (4 - 2) =$ $5x + 2$	In Step 2, Professor K combines like terms (terms with the same variable) by adding the ***coefficients*** (the multiplier of the variable: the number in front of the variable). Professor K determines that $5x + 2$ is the binomial difference of $(8x + 4)$ and $(3x + 2)$.

Example Set 2 shows how Professor K subtracts two more binomials.

Example Set 2

Subtract the following binomials.

$(6x + 2) - (2x + 1)$

Step 1	$6x + 2 - 2x - 1 =$ $6x - 2x + 2 - 1$	Professor K begins to determine the difference of the binomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $a + b = b + a$.
Step 1a	$-(2x + 1) = -1(2x + 1) =$ $-1 \bullet 2x + (-1) \bullet 1$ $-2x - 1$	In Step 1a, Professor K shows you how he used the Distributive Property to distribute the negative sign within the parentheses of the expression, $-(2x + 1)$. A -1 is understood to be in front of the parentheses. Therefore, Professor K distributes the -1 into $2x + 1$ by multiplying each term times -1. This changes all the signs to obtain $-2x - 1$. Professor K used this answer in Step 1 for the rearrangement of the terms to complete the operation called for.
Step 2	$6x - 2x + 2 - 1 =$ $(6 - 2)x + (2 - 1) =$ $4x + 1$	In Step 2, Professor K combines like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $4x + 1$ is the binomial difference of $(6x + 2)$ and $(2x + 1)$.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Example Set 3 shows how Professor K subtracts a binomial from a binomial.

Example Set 3

Subtract the following binomials.

$(7x + 2) - (3x - 3)$

Step 1	$7x + 2 - 3x + 3 =$ $7x - 3x + 2 + 3$	Professor K begins to determine the difference of the binomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $a + b = b + a$.
Step 1a	$-(3x - 3) = -1(3x - 3) =$ $-1 \bullet 3x + (-1) \bullet (-3)$ $-3x + 3$	In Step 1a, Professor K shows you how he used the Distributive Property to distribute the negative sign within the parentheses of the expression, $-(3x - 3)$. A -1 is understood to be in front of the parentheses. Therefore, Professor K distributes the -1 into $3x - 3$ by multiplying each term times -1. This changes all the signs to obtain $-3x + 3$. Professor K used this answer in Step 1 for the rearrangement of the terms to complete the necessary operation.
Step 2	$7x - 3x + 2 + 3 =$ $(7 - 3)x + (2 + 3) =$ $4x + 5$	In Step 2, Professor K combines like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $4x + 5$ is the binomial difference of $(7x + 2)$ and $(3x - 3)$.

You can subtract various other polynomials by following the same steps as in these examples. Show Professor K that you understand how to subtract binomials by working the activities for this lesson.

LIFE PRINCIPLE



"The key to a person's dreams lies within."

—George Washington Carver
(c. 1864-1943)
Botanist and scientist

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

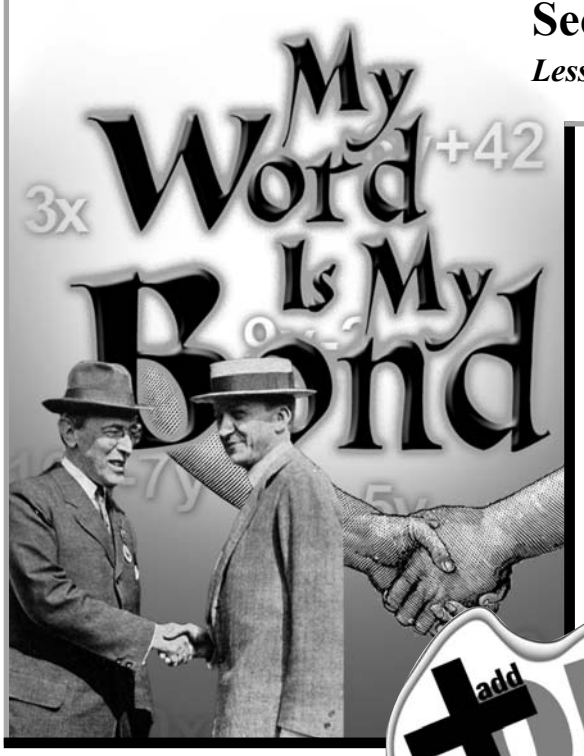
Lesson 13

Algebra

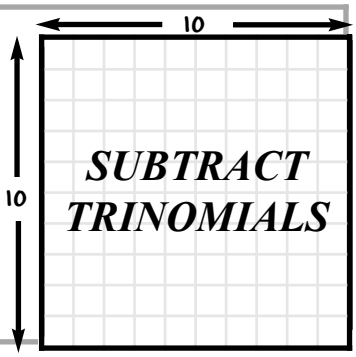
Chapter 5

Section 3

Lessons 11-15







In the previous lesson you learned that subtraction is the opposite of addition even when dealing with ***polynomials***. Not only can you add binomials and trinomials, you can also subtract these polynomials. Look at **Example Set 1** as Professor K subtracts a binomial and a trinomial.

Example Set 1

Subtract the following polynomials.
$(15x - 7) - (5x - y - 8)$

Step 1 $15x - 7 - 5x + y + 8 =$
$15x - 5x + y - 7 - 8$

Professor K begins to determine the difference of the polynomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $a + b = b + a$.

Step 1a $-(5x - y - 8) =$
$-1(5x - y - 8)$
$-1 \bullet 5x - (-1) \bullet y - (-1) \bullet 8$

In Step 1a, Professor K shows you how he used the Distributive Property to distribute the negative sign within the parentheses of the expression, $-(5x - y - 8)$. A -1 is understood to be in front of the parentheses. Therefore, Professor K distributes the -1 into $5x - y - 8$ by multiplying each term by -1. This changes all the signs to obtain $-5x + y + 8$. Professor K used the answer from Step 1 for the rearrangement of the terms to complete the needed operation.

VOCABULARY

Polynomial: an algebraic expression consisting of more than one term; may involve many terms



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Step 2	$15x - 5x + y - 7 + 8 =$ $(15 - 5)x + y + (-7 + 8)$ $10x + y + 1$	In Step 2, Professor K combines like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $10x + y + 1$ is the trinomial difference of $(15x - 7)$ and $(5x - y - 8)$.

Example Set 2 shows how Professor K subtracts a binomial from a trinomial.

Example Set 2

Subtract the following polynomials.

$(2x + 3y + 5) - (x - 6)$

Step 1	$2x + 3y + 5 - x + 6 =$ $2x - x + 3y + 5 + 6$	Professor K begins to determine the difference of the binomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $a + b = b + a$.
Step 1a	$-(x - 6) = -1(x - 6)$ $-1 \bullet x - (-1) \bullet (6)$ $-x + 6$	In Step 1a, Professor K shows you how he used the Distributive Property to distribute the negative sign within the parentheses of the expression, $-(x - 6)$. A -1 is understood to be in front of the parentheses. Therefore, Professor K distributes the -1 into $x - 6$ by multiplying each term by -1. This changes all the signs to obtain $-x + 6$. Professor K used this answer in Step 1 for the rearrangement of the terms to complete the needed operation.
Step 2	$2x - x + 3y + 5 + 6 =$ $(2 - 1)x + 3y + (5 + 6)$ $x + 3y + 11$	In Step 2, Professor K combines like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $x + 3y + 11$ is the trinomial difference of $(2x + 3y + 5)$ and $(x - 6)$.

You can subtract various other polynomials by following the same steps as in these examples. Show Professor K that you understand how to subtract binomials and trinomials by working the activities for this lesson.

LIFE PRINCIPLE



"There is no pillow so soft as a clear conscience."

—French Proverb

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



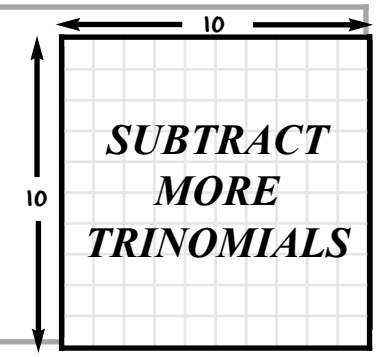
Lesson 14

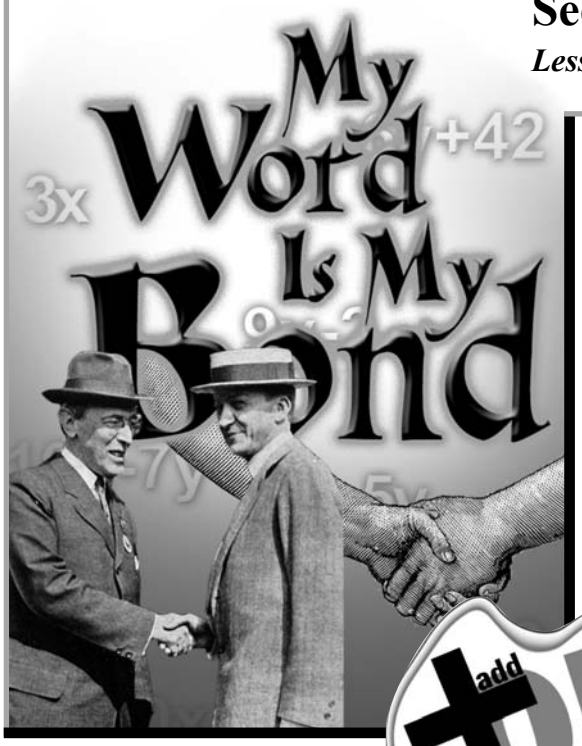
Algebra

Chapter 5

Section 3

Lessons 11-15





Trinomials can be subtracted as well as added. In this lesson, Professor K will show you how to perform the operation of subtraction on two ***trinomials.***



Professor K

In **Example Set 1**, Professor K shows you how to subtract two trinomials.



Example Set 1

Subtract the following trinomials.

$(5a + 12b + 8c) - (2a + 5b + 5c)$



$5a + 12b + 8c - 2a - 5b - 5c =$
$5a - 2a + 12b - 5b + 8c - 5c$

Professor K begins to determine the difference of the trinomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $a + b = b + a$.



Trinomial: a polynomial consisting of three terms

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Step 1a $-(2a + 5b + 5c) =$
$-1(2a + 5b + 5c)$
$-1 \bullet 2a + (-1) \bullet 5b + (-1) \bullet 5c$
$-2a - 5b - 5c$

In Step 1a, Professor K shows you how he used the Distributive Property to distribute the negative sign within the parentheses of the expression, $-(2a + 5b + 5c)$. A -1 is understood to be in front of the parentheses. Therefore, Professor K distributes the -1 into $2a + 5b + 5c$ by multiplying each term by -1. This changes all the signs to obtain $-2a - 5b - 5c$. Professor K used this answer in Step 1 for the rearrangement of the terms to complete the needed operation.

Step 2 $(5 - 2)a + (12 - 5)b + (8 - 5)c =$
$3a + 7b + 3c$

In Step 2, Professor K combines like terms (terms with the same variable) by subtracting the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $3a + 7b + 3c$ is the trinomial difference of $(5a + 12b + 8c) - (2a + 5b + 5c)$.

In **Example Set 2**, Professor K shows you how to subtract two trinomials.

Example Set 2

Subtract the following trinomials.
$(6x + 2y + 4) - (2x + y + 3)$

Step 1 $6x + 2y + 4 - 2x - y - 3 =$
$6x - 2x + 2y - y + 4 - 3$

Professor K begins to determine the difference of the trinomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $a + b = b + a$.

Step 1a $-(2x + y + 3) = -1(2x + y + 3)$
$-1 \bullet 2x + (-1) \bullet y + (-1) \bullet 3$
$-2x - y - 3$

In Step 1a, Professor K shows you how he used the Distributive Property to distribute the negative sign within the parentheses of the expression, $-(2x + y + 3)$. A -1 is understood to be in front of the parentheses. Therefore, Professor K distributes the -1 into $2x + y + 3$ by multiplying each term by -1. This changes all the signs to obtain $-2x - y - 3$. Professor K used this answer in Step 1 for the rearrangement of the terms to complete the needed operation.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Step 2	$(6 - 2)x + (2 - 1)y + (4 - 3) = 4x + y + 1$	In Step 2, Professor K combines like terms (terms with the same variable) by subtracting the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $4x + y + 1$ is the trinomial difference of $(6x + 2y + 4) - (2x + y + 3)$.

You can subtract various other polynomials by following the same steps as in these examples. Show Professor K that you understand how to subtract trinomials by working the activities for this lesson.

LIFE PRINCIPLE



"Perseverance is not a long race; it is many short races one after another."

—Walter Elliott

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



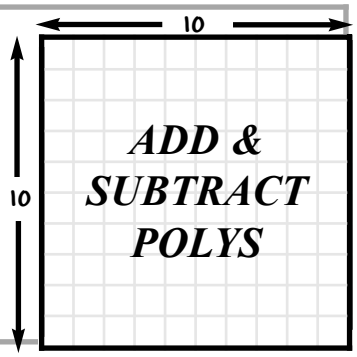
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Lesson 15

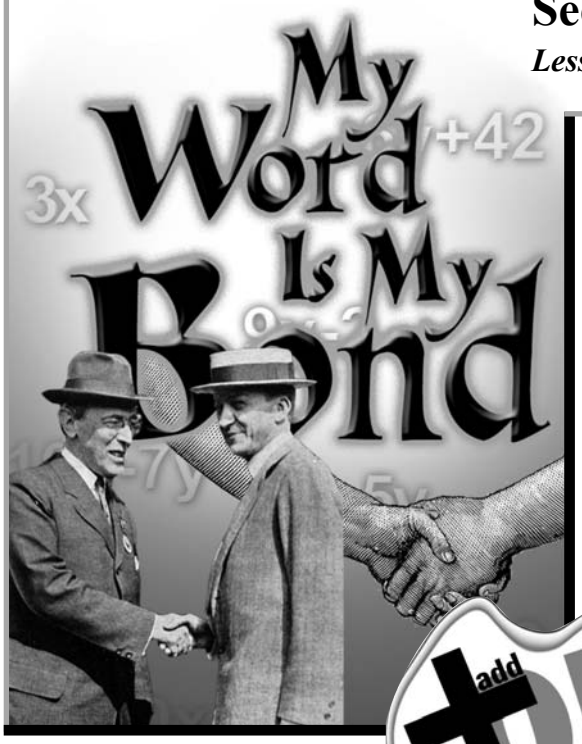
Algebra

Chapter 5



Section 3

Lessons 11-15



Polynomials containing more than three terms are added or subtracted in the same manner as polynomials of fewer components—the process just takes a little longer to ***execute***. Professor K will show you in this lesson how to accomplish these operations on ***polynomials of four terms***. Look at **Example Set 1** as Professor K adds two polynomials of four terms each.





Example Set 1

Add the following polynomials.

$(8x^3 + 3x^2 + x + 4) +$
$(3x^3 + 2x^2 + 4x + 2)$

Step 1

$8x^3 + 3x^2 + x + 4 + 3x^3 + 2x^2 + 4x + 2 =$
$8x^3 + 3x^3 + 3x^2 + 2x^2 + x + 4x + 4 + 2$

Professor K begins to determine the difference of the binomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $a + b = b + a$.

VOCABULARY

Execute: to perform, accomplish, carry out or finish a pre-planned activity
Polynomials of four terms: a polynomial containing four terms

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Step 2

$8x^3 + 3x^3 + 3x^2 + 2x^2 + x + 4x + 4 + 2 =$

$(8 + 3)x^3 + (3 + 2)x^2 + (1 + 4)x + (4 + 2) =$

$11x^3 + 5x^2 + 5x + 6$

In Step 2, Professor K combines like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $11x^3 + 5x^2 + 5x + 6$ is the sum of the two polynomials $(8x^3 + 3x^2 + x + 4)$ and $(3x^3 + 2x^2 + 4x + 2)$.

Example Set 2 shows how Professor K subtracts a polynomial of four terms from a polynomial of four terms. Notice that the polynomials are the same as in **Example Set 1**, the only difference is the operation: subtraction, not addition.

Example Set 2

Subtract the following polynomials.

$(8x^3 + 3x^2 + x + 4) - (3x^3 + 2x^2 + 4x + 2)$

Step 1

$8x^3 + 3x^2 + x + 4 - 3x^3 - 2x^2 - 4x - 2 =$

$8x^3 - 3x^3 + 3x^2 - 2x^2 + x - 4x + 4 - 2$

Professor K begins to determine the difference of the binomials by dropping the parentheses and rearranging the terms. (See Step 1.) Remember, terms can be rearranged because of the Commutative Property, which states that $a + b = b + a$.

Step 1a

$- (3x^3 + 2x^2 + 4x + 2) =$

$-1(3x^3 + 2x^2 + 4x + 2)$

$-1(3x^3) + -1(2x^2) + -1(4x) + -1(2)$

$-3x^3 - 2x^2 - 4x - 2$

In Step 1a, Professor K shows you how he used the Distributive Property to distribute the negative sign within the parentheses of the expression, $-(3x^3 + 2x^2 + 4x + 2)$. A -1 is understood to be in front of the parentheses. Therefore, Professor K distributes the -1 into $3x^3 + 2x^2 + 4x + 2$ by multiplying each term by -1. This changes all the signs to obtain $-3x^3 - 2x^2 - 4x - 2$. Professor K used this answer in Step 1 for the rearrangement of the terms to complete the necessary operation.

Step 2

$8x^3 - 3x^3 + 3x^2 - 2x^2 + x - 4x + 4 - 2 =$

$(8 - 3)x^3 + (3 - 2)x^2 + (1 - 4)x + (4 - 2) =$

$5x^3 + x^2 - 3x + 2$

In Step 2, Professor K combines like terms (terms with the same variable) by adding the coefficients (the multiplier of the variable: the number in front of the variable). Professor K determines that $5x^3 + x^2 - 3x + 2$ is the difference of the polynomials $8x^3 + 3x^2 + x + 4$ and $3x^3 + 2x^2 + 4x + 2$.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

You can add and subtract various other polynomials by following the same steps as in these examples. Show Professor K that you understand how to add and subtract polynomials containing more than three terms by working the activities for this lesson.

Life Principle



"*First mend yourself, and then mend others.*"

—Jewish Proverb

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Intermediate Math Skills

Mathematics Charts

LENGTH - ENGLISH SYSTEM

12 inches	=	1 foot
3 feet	=	1 yard
5.5 yards	=	1 rod
320 rods	=	1 mile
1,760 yards	=	1 mile
5,280 feet	=	1 mile

VOLUME - ENGLISH SYSTEM

1 tablespoon, Tbsp, T	=	3 teaspoons, tsp
1 cup, c	=	16 tablespoons, Tbsp, T
1 pint, pt	=	2 cups, c
1 quart, qt	=	2 pints, pts
1 gallon, gal	=	4 quarts, qts

WEIGHT - ENGLISH SYSTEM

1 pound, lb	=	16 ounces, oz
1 hundredweight, cwt	=	100 pounds, lbs
1 ton	=	20 hundredweight, cwt
1 ton	=	2000 pounds, lbs

LENGTH - METRIC SYSTEM

1 meter	=	1,000 millimeters
1 meter	=	100 centimeters
1 kilometer	=	1,000 meters
1 centimeter	=	10 millimeters

VOLUME - METRIC SYSTEM

1 liter	=	1,000 milliliters
1 liter	=	100 centiliters
1 kiloliter	=	1,000 liters
1 centiliter	=	10 milliliters

WEIGHT - METRIC SYSTEM

1 gram	=	1,000 milligrams
1 gram	=	100 centigrams
1 kilogram	=	1,000 grams
1 centigram	=	10 milligrams

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

Mathematics Charts

PERFECT SQUARES

$\sqrt{1}=1$	$\sqrt{25}=5$	$\sqrt{81}=9$	$\sqrt{169}=13$
$\sqrt{4}=2$	$\sqrt{36}=6$	$\sqrt{100}=10$	$\sqrt{196}=14$
$\sqrt{9}=3$	$\sqrt{49}=7$	$\sqrt{121}=11$	$\sqrt{225}=15$
$\sqrt{16}=4$	$\sqrt{64}=8$	$\sqrt{144}=12$	$\sqrt{256}=16$

PERFECT CUBES

$\sqrt[3]{1}=1$	$\sqrt[3]{-8}=-2$	$\sqrt[3]{64}=4$	$\sqrt[3]{-125}=-5$
$\sqrt[3]{-1}=-1$	$\sqrt[3]{27}=3$	$\sqrt[3]{-64}=-4$	$\sqrt[3]{216}=6$
$\sqrt[3]{8}=2$	$\sqrt[3]{-27}=-3$	$\sqrt[3]{125}=5$	$\sqrt[3]{-216}=-6$

$$P = \frac{\text{the number of elements in the set containing favorable outcomes}}{\text{the number of elements in the sample set}}$$

$$\textit{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

FORMULAS

Surface Area	Area	Volume	Perimeter/ Circumference	Others
Sphere: $A_s = 4\pi r^2$	**Triangle:** $A = {}^1\!/_2 bh$	**Sphere:** $V = {}^4\!/_3 \pi r^3$	**Perimeter of a Rectangle:** $P = 2w + 2l$	**Pythagorean Theorem:** $c^2 = a^2 + b^2$
Cylinder: $A_s = 2\pi rh + 2\pi r^2$	**Rectangle:** $A = l \times w$	**Cone:** $V = {}^1\!/_3 \pi r^2 h$	**Circumference of a Circle:** $C = 2\pi r$ or $C = \pi d$	**Convert to Fahrenheit:** $F = {}^9\!/_5 C + 32$
Cube: $A_s = 6s^2$	**Circle:** $A = \pi r^2$	**Rectangular Prism:** $V = l \times w \times h$		**Convert to Celsius:** $C = {}^5\!/_9 (F - 32)$
Lateral Surface Area of a Cone: $A_{ls} = \pi rl$	**Trapezoid:** $A = {}^1\!/_2 (b_1 + b_2)h$	**Square Pyramid:** $V = ({}^1\!/_3) lwh$		**Distance:** $d = r \bullet t$
		Triangular Prism: $V = {}^1\!/_2 bhl$		**Slope:** $y = mx = b$
		Cylinder: $V = \pi r^2 h$		**Simple Interest:** $I = prt$

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

TIME CONVERSIONS

1 week = 7 days
1 year = 12 months

CONVERSIONS

English to Metric	
1 ounce (oz)	28.35 grams (g)
1 pound (lb)	454 grams (g)
1 ton (T)	907.2 kilograms (kg)

Metric to English	
1 gram (g)	.035 ounce (oz)
1 kilogram (kg)	2.2 pounds (lb)
1 metric ton (mt)	2200 pounds (lb)

English to Metric	
1 cubic inch	16.39 cubic cm
1 gallon	3.785 liters
1 cubic foot	28.316 liters
1 quart	.946 liters

Metric to English	
1 cubic cm	.061 cubic inch
1 cubic meter	35.315 cubic feet
1 liter	1.05 quarts
1 liter	.264 gallon

English to Metric	
1 inch	2.54 cm
1 foot	0.3048 meter
1 yard	0.9144 meter
1 mile	1.6 km

Metric to English	
1 centimeter	0.3937 in.
1 meter	39.37 in.
1 meter	1.09 yds
1 kilometer	0.6213 mile

AREA CONVERSIONS

1 square centimeter = 0.155000 sq in.
1 square meter = 10.763910 sq ft
1 square inch = 645.16 sq mm
1 square foot = 0.09290304 sq meter
1 square yard = 0.836127 sq meter
1 square foot = 144 sq in.

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.